Frobenius reciprocity: symplectic, prequantum, diffeological*

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Abstract: Ratiu–Z.[†] established "Frobenius reciprocity" as a bijection *t* between certain symplectically reduced spaces (which need not be manifolds), and conjectured:

• *t* is a diffeomorphism, relative to the subquotient diffeologies of these spaces;

• *t* respects the reduced diffeological 2-forms they may (or might not) carry. We *prove this*, and give *new sufficient conditions* for the reduced forms to exist.

*arXiv:2403.3927, joint with Gabriele Barbieri and Jordan Watts. †arXiv:2007.9434, building on ideas of Guillemin–Sternberg (1983).

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Let (X, ω, Φ) be a Hamiltonian G-space (G: Lie group, Φ : equivariant moment map). The *reduced space*

 $\mathrm{X}/\!\!/\mathrm{G}:=\Phi^{-1}(0)/\mathrm{G}$

eed not be a manifold; but it has a natural ("subquotient") *diffeology*. may or might not carry a reduced 2-form:

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Definition

We say that X//G carries a reduced 2-form if there is a (diffeological) 2-form $\omega_{X//G}$ such that $j^*\omega = \pi^*\omega_{X//G}$, where

$$\begin{array}{ccc} \Phi^{-1}(\mathbf{0}) & \stackrel{j}{\longrightarrow} X \\ & \pi \\ & \chi \\ X /\!\!/ \mathbf{G}. \end{array}$$

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Note: we will see that if $\omega_{X//G}$ exists, then it is unique (and closed).

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Important special case (which sounds more general):

Example: $Hom_G(X_1, X_2)$ (Guillemin-Sternberg 1982)

This is

 $Hom_G(X_1, X_2) := (X_1^- \times X_2) /\!\!/ G$

where (X_i, ω_i, Φ_i) are Hamiltonian G-spaces and $X^- := (X, -\omega, -\Phi)$ So the product here has diagonal G-action, 2-form $\omega_2 - \omega_1$, and moment map $\Phi(x_1, x_2) = \Phi_2(x_2) - \Phi_1(x_1)$.

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Note that \heartsuit boils down to X₂//G when (X₁, ω_1 , Φ_1) = ({0}, 0, 0); so asking when it carries a reduced 2-form includes the original question about ϕ .

More generally, Guillemin–Sternberg took for X_1 a coadjoint orbit $G(\mu)$, and noted that \heartsuit then boils down to the space $\Phi_2^{-1}(\mu)/G_0$ of Marsden–Weinstein: this is their famous "shifting trick".

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) Covering. All constant maps $\mathbb{R}^n \to X$ are in \mathbb{P} , for all n.

12) Locality. Let V → X be a map with V ∈ x_n. If every point of V has an open neighborhood U such that P_{IU} ∈ P, then P ∈ P.

Smooth compatibility: Let $\bigcup \stackrel{\sim}{\rightarrow} V \stackrel{\sim}{\rightarrow} X$ be maps with $(U, V) \in \mathbb{R}_{+} \times \mathfrak{r}_{0}$. If $\mathbb{P} \in \mathcal{P}$ and $\psi \in \mathbb{C}^{\infty}(U, V)$, then $\mathbb{P} \circ \psi \in \mathcal{P}$.

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- D2) Locality. Let $V \xrightarrow{P} X$ be a map with $V \in \tau_n$. If every point of V has an open neighborhood U such that $P_{|U} \in \mathcal{P}$, then $P \in \mathcal{P}$.
- D3) Smooth compatibility. Let $U \xrightarrow{\psi} V \xrightarrow{P} X$ be maps with $(U, V) \in \tau_m \times \tau_n$. If $P \in P$ and $\psi \in C^{\infty}(U, V)$, then $P \circ \psi \in P$.

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Let X be a set. A *diffeology* on X is a subset \mathcal{P} of $\bigcup_{n \in \mathbb{N}, U \in \tau_n} \text{Maps}(U, X)$ satisfying (D1–D3). We call its members with domain $U \in \tau_n$, *n*-plots.

A map $(X, \mathcal{P}) \xrightarrow{r} (Y, \Omega)$ of diffeological spaces (: sets with diffeologies) is called *smooth* if $P \in \mathcal{P}$ implies $F \circ P \in \Omega$.

If $(X, \mathcal{P}) \stackrel{\mu}{\rightarrow} (X, \Omega)$ is smooth, i.e. $\mathcal{P} \subset \Omega$, we call \mathcal{P} *finer* and Ω *coarser*.

E.g.: {locally constant maps} =: $\mathcal{P}_{discrete} \subset \mathcal{P} \subset \mathcal{P}_{coarse} := \{all maps\}$

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Let X be a set. A *diffeology* on X is a subset \mathcal{P} of $\bigcup_{n \in \mathbb{N}, U \in \tau_n} Maps(U, X)$ satisfying (D1–D3). We call its members with domain $U \in \tau_n$, *n*-plots.

A map $(X, \mathcal{P}) \xrightarrow{F} (Y, \mathcal{Q})$ of diffeological spaces (: sets with diffeologies) is called *smooth* if $P \in \mathcal{P}$ implies $F \circ P \in \mathcal{Q}$.

If $(X, \mathcal{P}) \xrightarrow{id} (X, \Omega)$ is smooth, i.e. $\mathcal{P} \subset \Omega$, we call \mathcal{P} *finer* and Ω *coarser*.

E.g.: {locally constant maps} =: $\mathcal{P}_{discrete} \subset \mathcal{P} \subset \mathcal{P}_{coarse} := {all maps}.$

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So every manifold has a canonical diffeology. But also:

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So every manifold has a canonical diffeology. But also:

Let Y be a diffeological space and *i* : X → Y an injection. Then X has a coarsest diffeology making *i* smooth, the *subset diffeology*. Its plots are the maps P : U → X such that *i* ∘ P is a plot of Y.

Universal property: A map F to X is smooth iff $i \circ$ F is smooth.

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Let X be a diffeological space and $s \in X \rightarrow X$ a surjection. Then X has a finest diffeology making a smooth, the quotient diffeology. Its n-plots are the maps $Q \in U \rightarrow X$ that have around each $u \in U$ a local lift; an n-plot $R \in V \rightarrow X$ with $u \in V \subset U$ and $O_0 = s$ o R.

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Universal property: A map F from Y is smooth iff F o s is smooth

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• The promised *subquotient diffeology* of $X/\!\!/G = \Phi^{-1}(0)/G$ results:

take subset diffeology on $\Phi^{-1}(0)$, then quotient — or equivalently, as one can show, take quotient diffeology on X/G, then subset.

• Any map F : $X \rightarrow Y$ between diffeological spaces can be factored

$$egin{array}{cccc} & {
m X} & {
m F} & {
m Y} \ & s & & {
m \hat{f}} & & {
m \hat{f}} \ & & {
m \hat{f}} & & {
m \hat{f}} \ & X/\sim & {
m \dot{\dot{F}}} & {
m F}({
m X}), \end{array} egin{array}{ccccc} & {
m Y} & & {
m \hat{F}} & {
m \hat{F}} \ & {
m \hat{F}} & {
m \hat{F}} \ & {
m \hat{F}} \end{array}$$

where s = quotient map by the equivalence relation 'F(x_1) = F(x_2)', $\dot{F} =$ bijection of that quotient with F(X), i = inclusion of that image into Y. With quotient (resp. subset) diffeology on X/ \sim (resp. F(X)), the universal properties we saw imply: F smooth \Leftrightarrow F smooth.

Definitions

F is strict if both F and F⁻¹ are smooth (i.e., F is a diffeomorphism).
An induction is a strict injection. Example: inclusion of a subset.
A subduction is a strict surjection. Example: projection to a quotient

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m X} & & \stackrel{
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m Y} \ {
m s} & & & & \uparrow i \ {
m X}/\sim & \stackrel{
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m F}({
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• The promised *subquotient diffeology* of $X/\!/G = \Phi^{-1}(0)/G$ results: take subset diffeology on $\Phi^{-1}(0)$, then quotient — or *equivalently*, as one can show, take quotient diffeology on X/G, then subset.

- Any map $F:X\to Y$ between diffeological spaces can be factored

$$egin{array}{ccc} X & \longrightarrow & Y \ s & & & \uparrow i \ X/\sim & \stackrel{\dot{\mathrm{F}}}{\longrightarrow} & \mathrm{F}(\mathrm{X}), \end{array} egin{array}{ccc} \mathrm{F} & \mathrm{F} \circ s, \ \mathrm{F} = i \circ \dot{\mathrm{F}} \circ s, \end{array}$$

where s = quotient map by the equivalence relation 'F(x_1) = F(x_2)', $\dot{F} =$ bijection of that quotient with F(X), i = inclusion of that image into Y. With quotient (resp. subset) diffeology on X/ \sim (resp. F(X)), the universal properties we saw imply: F smooth $\Leftrightarrow \dot{F}$ smooth.

- F is *strict* if both \dot{F} and \dot{F}^{-1} are smooth (i.e., \dot{F} is a *diffeomorphism*).
- An induction is a strict injection. Example: inclusion of a subset.
- A *subduction* is a strict surjection. *Example*: projection to a quotient.

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Interlude: a historical question.

I have seen "strict" defined for

{topological groups, continuous morphisms}: Bourbaki 1960,

. {diffeological groups, smooth morphisms}: Donato 1984,

(diffeological spaces, smooth maps): Souriau 1985,

(1) (topological spaces, continuous maps): Bourbald 2016.

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Question

Is 4. really nowhere to be found before 1985?

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§7. Frobenius reciprocity Let us call *ordinary* the *k*-forms on Euclidean open sets and operations on them (pull-back, exterior derivative).

Definitions (Diffeological Cartan-de Rham calculus)

Let X and Y be diffeological spaces.

A k-form α on Y is a functional which sends each plot $\mathbb{P}: \mathbb{V} \to \infty$ an ordinary k-form on V, denoted $\mathbb{P}^*\alpha$.

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 $(P\circ \varphi)^* \alpha = \varphi^* P^* \alpha, \qquad \varphi^*: \text{ ordinary pull-back}.$

supported bases \mathbb{R}^n along a support map $\mathbb{P}: \mathbb{R} \to \mathbb{R}$ is the k-form on \mathbb{R} and the form on \mathbb{R} is a plot of \mathbb{R} (so \mathbb{P} or \mathbb{P} is a plot of \mathbb{R}), then

 $\mathbb{P}^*\mathbb{P}^* \mathfrak{a} = (\mathbb{P} \circ \mathbb{P})^* \mathfrak{a}, \dots, \mathbb{P}^*$; being defined.



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 $(\mathbf{P} \circ \phi)^* \alpha = \phi^* \mathbf{P}^* \alpha, \qquad \phi^*$: ordinary pull-back.

Its *pull-back* $F^*\alpha$ by a smooth map $F : X \to Y$ is the *k*-form on X defined by: if P is a plot of X (so $F \circ P$ is a plot of Y), then

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 $P^*F^*\alpha = (F \circ P)^*\alpha$, F^* : being defined.

 Its exterior derivative dα is the (k + 1)-form defined for all pl P of Y by P^{*} dα = dP^{*}α, with ordinary d on the right-hand side.

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Comments on the proof. Necessity is clear: if $\alpha = s^*\beta$, we have

 $P^* \alpha = P^* s^* \beta = (s \circ P)^* \beta,$ $Q^* \alpha = Q^* s^* \beta = (s \circ Q)^* \beta$

by definition of s^* ; \diamond follows. Proving the rest takes about 2 pages. \Box

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There is a basic criterion for when a k-form descends to a quotient:

Theorem (Souriau's criterion, 1985)

et *s* : X → Y be a subduction, α a *k*-form on X. In order that $\alpha = s^*\beta$ *r* some β on Y, it is necessary and sufficient that all pairs of plots P, Q X satisfy

Moreover, β is then unique.

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All diffeological notions used in §1 have now been defined.

Existence of $\omega_{X/\!/G}$: Prior State of the Art

If the G-action on the level $C = \Phi^{-1}(0)$ is *locally free and proper*, it has long been known that X//G is an (effective) orbifold with an 'orbifold 2-form' (proof in Cushman–Bates 1997). Now when orbifolds are regarded as diffeological spaces, 'orbifold forms' define diffeological forms and conversely (Karshon–Watts 2016).

Briefly, §§4–6 will improve on this by showing: it suffices to assume *locally free* or proper, or strict.

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Existence of $\omega_{X/\!\!/G}$: Prior State of the Art

• If the G-action on the level $C = \Phi^{-1}(0)$ is *locally free and proper*, it has long been known that X//G is an (effective) orbifold with an 'orbifold 2-form' (proof in Cushman–Bates 1997). Now when orbifolds are regarded as diffeological spaces, 'orbifold forms' define diffeological forms and conversely (Karshon–Watts 2016).

Note: locally free means that the infinitesimal stabilizer g_{μ} is zero. for all $x \in C$. As $Im(D\Phi(x)) = annihilator(g_{\mu})$, it follows that 0 is a regular value, so C is a manifold.

Of course, if the G-action on C is free and proper, then X/G itself is a manifold with a symplectic 2-form (Marsden-Weinstein 1999)

Briefly, §§4–6 will improve on this by showing: it suffices to assume locally free or proper, or strict.

§3. Orbifolds

§1. Symplect reduction

§2. Diffeology

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Let a diffeological group G act on a diffeological space X. We consider the map

 $heta: \mathsf{G} imes \mathsf{X} o \mathsf{X} imes \mathsf{X}, \qquad heta(g,x) = (x,g(x)).$

Definition

The G-action is *strict* if θ is a strict map (§2).

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Theorem 1

In the setting of §1, suppose that the G-action on $C = \Phi^{-1}(0)$ is strict. Then X//G carries a reduced 2-form.

Comments on the proof. Our above "smooth division"

 $\mathsf{Q}(u) = \mathsf{R}(u)(\mathsf{P}(u))$

is just what's needed for a straightforward application of Souriau's criterion \diamond (using elementary properties of moment maps). Subtler results (§5, §6) tend to use \diamond in tandem with e.g. Baire category.

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Example: Ind_H^GY

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$Ind_{H}^{G}Y := (T^{*}G \times Y)/\!\!/H = \psi^{-1}(0)/H$

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 $egin{array}{rll} \phi(p,y) &=& pq^{-1} \ \phi(p,y) &=& \Psi(y) - q^{-1}p_{||_{
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 $(p\in \mathrm{T}_q^*\mathrm{G}).$

- When H is closed,
 is a Marsden–Weinstein reduced manifold, with a residual G-action and moment map Φ_{L//H} : Ind^G_H Y → g^{*}.
- When H is not closed, the H-action on φ⁻¹(0) is still *strict*: so Theorem 1 yields a reduced 2-form ω_{L//H}, and we have a "para symplectic" *induced Hamiltonian G-space* (Ind^G_H Y, ω_{L//H}, Φ_{L//H})

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§1. Symplection

Example: Ind_H^GY

This is

$$Ind_{H}^{G} Y := (T^{*}G \times Y) / H = \psi^{-1}(0) / H$$

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§6. Proper actions

§7. Frobenius reciprocity

where: G is a Lie group, H is an *arbitrary subgroup* (hence canonically also a Lie group: Bourbaki 1972), (Y, ω_Y, Ψ) is a Hamiltonian H-space, and $L := T^*G \times Y$ is the Hamiltonian $G \times H$ -space with action $(g, h)(p, y) = (gph^{-1}, h(y))$ and moment map $\phi \times \phi : L \to \mathfrak{g}^* \times \mathfrak{h}^*$,

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§3. Orbifolds
§4. Strict actions
§5. Locally free actions
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This is $(T^*G)//H$, the reduction of T^*G by the 'right' action of H.

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We can ask, then, if the equality survives. It does at least for dense H:

Theorem 2

Let G be a Lie group, H any dense subgroup. Then $(T^*G)/H = T^*(G/H)$ as diffeological, parasymplectic Hamiltonian G-spaces.

Example: G the 2-torus, H an irrational winding, $G/H = T_{\alpha}$

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Comments on the proof. A key step in Iglesias-Zemmour's definition is

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Theorem 3 (B. Clark–Z.)

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 $T_x^*(X) := \Omega^1(X) / \{1 \text{-forms vanishing at } x\}.$

For that we have, with $\Pi : G \to G/H$,

Proposition

For H dense in G, Π^* is a linear bijection $\Omega^1(G/H) \xrightarrow{\sim} \text{annihilator}(\mathfrak{h})$.

(Surjectivity is by another application of Souriau's \Diamond .) In fact, it is not hard to generalize this into

Theorem 3 (B. Clark-Z.)

Let G be a Lie group, H any dense subgroup. Then $\mathfrak{h} \subset \mathfrak{g}$ is an ideal, and

 $\Omega^{\bullet}(G/H) = \bigwedge^{\bullet}(\mathfrak{g}/\mathfrak{h})^*, \qquad H^{\bullet}_{dR}(G/H) = H^{\bullet}(\mathfrak{g}/\mathfrak{h})$

(Lie algebra cohomology à la Chevalley–Eilenberg).

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Theorem 4

In the setting of §1, suppose the G-action on $C = \Phi^{-1}(0)$ is **locally free** (i.e. all $x \in C$ have infinitesimal stabilizer $g_x = \{0\}$) and G is connected. Then X//G carries a reduced 2-form.

Comments on the proof. Under the hypotheses, standard properties of the moment map:

 $\operatorname{Ker}(\operatorname{D}\Phi(x)) = \mathfrak{g}(x)^{\circ}, \qquad \operatorname{Im}(\operatorname{D}\Phi(x)) = \operatorname{annihilator}(\mathfrak{g}_x).$

readily imply that 1) 0 is a regular value of Φ , 2) C is a submanifold, 3) in C the G-orbits are the leaves of a *foliation* \mathcal{F} , 4) $\omega_{|C} := j^* \omega$ is *basic* for \mathcal{F} , i.e., G-invariant with $\mathfrak{g}(x) \subset \operatorname{Ker}(\omega_{|C})$. A theorem on foliations by Hector *et al.* (2011) then implies the result.

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Recall that the action of a Lie group on a manifold is called *proper* if the map θ (§4) is proper, i.e., compact sets have compact preimages.

Theorem 5

In the setting of §1, suppose that the G-action on X is **proper**. (Example: G compact.) Then X//G carries a reduced 2-form $\omega_{X/G}$.

Comments on the proof. For proper actions, Sjamaar–Lerman–Bates (1991, 1997) showed that $X/\!/G = C/G$ is a 'stratified symplectic space', i.e. (among other things) a disjoint union of symplectic manifolds $(C_t/G, \omega_t)$ indexed by orbit types *t*. Our proof crucially uses the ω_t to show that $j^*\omega$ satisfies Souriau's criterion \Diamond . The resulting (*global*) $\omega_{X/\!/G}$ actually induces every ω_t , as the following corollary states.

Corollary

In Theorem 5, $\omega_{X/\!/G}$ restricts to the Sjamaar–Lerman–Bates ω_t on each reduced piece C_t/G .

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§7. Frobenius reciprocity

Let G be a Lie group, H a closed subgroup, X a Hamiltonian G-space, Y a Hamiltonian H-space. Recall Hom_G and Ind_H^G , and define Res_H^G .

§7. Frobenius reciprocity

There is a (diffeological) diffeomorphis

 $t: \operatorname{Hom}_{G}(X, \operatorname{Ind}_{H}^{G} Y) \to \operatorname{Hom}_{H}(\operatorname{Res}_{H}^{G} X, Y).$

Moreover, if one side carries a reduced 2-form, then so does the other, and t maps one form to the other.

Sketch of proof. The sides are respectively (M//H)//G and N//H, where

 $M = X^- \times T^*G \times Y$, resp. $N = X^- \times Y$

have $G \times H$ -action $(g, h)(x, p, y) = (g(x), gph^{-1}, h(y))$, resp. diagonal H-action, plus appropriate 2-forms ω_M and ω_N and moment maps

 $\phi_{M} \times \phi_{M} : M \to \mathfrak{g}^{*} \times \mathfrak{h}^{*}, \quad \text{resp.} \quad \phi_{N} : N \to \mathfrak{h}^{*}.$ Define $r : M \to N$ by $r(x, p, y) = (q^{-1}(x), y)$ for $p \in T^{*}_{\sigma}G$,

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§7. Frobenius reciprocity

Theorem 6

There is a (diffeological) diffeomorphism

 $t: \operatorname{Hom}_{G}(X, \operatorname{Ind}_{H}^{G} Y) \to \operatorname{Hom}_{H}(\operatorname{Res}_{H}^{G} X, Y).$

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Moreover, if one side carries a reduced 2-form, then so does the other, and t maps one form to the other.

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§6. Proper actions

§7. Frobenius reciprocity

Let G be a Lie group, H a closed subgroup, X a Hamiltonian G-space, Y a Hamiltonian H-space. Recall Hom_{G} and Ind_{H}^{G} , and define Res_{H}^{G} .

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where the *j*'s and π 's are inclusions and projections as in §1. One checks that *r* sends $(\phi_M \times \psi_M)^{-1}(0)$ to $\psi_N^{-1}(0)$, so there is a map *s* as indicated; and *s* sends G × H-orbits to H-orbits, so there is a map *t*. Likewise one checks that the right inverse $r' : N \to M$ defined by

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 $r'(x, y) = (x, \Phi(x), y)$ (where we identify $\mathfrak{g}^* \cong T_e^*G$) descends to an inverse t^{-1} of t. Now r and r' are quite explicitly smooth. Using the universal properties of inductions (the j's) and subductions (the π 's),

Next, assume that both sides carry reduced 2-forms, $\omega_{(M/H)//G}$ and $\omega_{N//H}$. We must prove $\omega_{(M//H)//G} = t^* \omega_{N//H}$. By chasing the diagram, one checks that this is equivalent to, not quite $\omega_M = r^* \omega_N$ but

$$j^* j_1^* \omega_{\rm M} = j^* j_1^* r^* \omega_{\rm N},$$
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an equality of 2-forms on $(\phi_M \times \phi_M)^{-1}(0)$ (usually not a manifold). Now (*) means that its sides coincide after pull-back by any plot P of that subset, i.e., by any *smooth map* P : U \rightarrow M *taking values in* $(\phi_M \times \phi_M)^{-1}(0)$. This is true, and can be checked in about 10 lines. Finally, assume merely that *one* reduced form exists, $\omega_{(M/H)/G}$ or $\omega_{M/H}$. Then we can *define* the other by $\omega_{(M/H)/G} = t^* \omega_M v_H$; and an

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 $r'(x, y) = (x, \Phi(x), y)$ (where we identify $\mathfrak{g}^* \cong T_e^*G$) descends to an inverse t^{-1} of t. Now r and r' are quite explicitly smooth. Using the universal properties of inductions (the j's) and subductions (the π 's), one deduces without trouble that t and t^{-1} are smooth, as claimed.

Next, assume that both sides carry reduced 2-forms, $\omega_{(M/H)//G}$ and $\omega_{N//H}$. We must prove $\omega_{(M/H)//G} = t^* \omega_{N//H}$. By chasing the diagram, one checks that this is equivalent to, not quite $\omega_M = \tau^* \omega_N$ but

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an equality of 2-forms on $(\phi_M \times \psi_M)^{-1}(0)$ (usually not a manifold). Now (*) means that its sides coincide after pull-back by any plot P of that subset, i.e., by any *smooth map* P : U \rightarrow M *taking values in* $(\phi_M \times \psi_M)^{-1}(0)$. This is true, and can be checked in about 10 lines. Finally, assume merely that *one* reduced form exists, $\omega_{(M/H)/G}$ or $\omega_{N/H}$. Then we can *define* the other by $\omega_{(M/H)/G} = t^* \omega_{N/H}$; and an easy chase using again (*) shows that it is indeed a reduced form.

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Next, assume that both sides carry reduced 2-forms, $\omega_{(M/H)//G}$ and $\omega_{N//H}$. We must prove $\omega_{(M//H)//G} = t^* \omega_{N//H}$. By chasing the diagram, one checks that this is equivalent to, not quite $\omega_M = r^* \omega_N$ but

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Final remark: Everything we have said can be adapted to work also in the category {*prequantum G-spaces*}, which more closely mirrors the motivating category {unitary representations}. For details, see arXiv:2007.9434 and arXiv:2403.3927.

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End!