Diffeological Reduced Spaces and Frobenius Reciprocity*

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Abstract: Ratiu-Z.^{\dagger} established "Frobenius reciprocity" as a bijection *t* between certain symplectically reduced spaces (which need not be manifolds), and conjectured:

• *t* is a diffeomorphism, relative to the subquotient diffeologies of these spaces;

• t respects the reduced diffeological 2-forms they may (or might not) carry. We prove this, and give new sufficient conditions for the reduced forms to exist.

*arXiv: 2403.3927, joint with Gabriele Barbieri and Jordan Watts.
*arXiv: 2007.9434, building on ideas of Guillemin-Sternberg (1983).

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§2. Diffeology

§3. Orbifolds

§4. Strict actions

§5. Locally free actions

§6. Proper actions

§7. Frobeniu reciprocity

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Let (X, ω, Φ) be a Hamiltonian G-space (G: Lie group, Φ : equivariant moment map). The *reduced space*

 $X/\!/G := \Phi^{-1}(0)/(0)$

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Definition

We say that X//G *carries a reduced* 2-*form* if there is a (diffeological) 2-form $\omega_{X//G}$ such that $j^*\omega = \pi^* \omega_{X//G}$, where

$$\begin{array}{ccc} \Phi^{-1}(\mathbf{0}) & \stackrel{j}{\longrightarrow} X \\ & \pi \\ & \chi /\!\!/ \mathbf{G}. \end{array}$$

Note: we will see that if $\omega_{X/\!/G}$ exists, then it is unique (and closed)

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Important special case (which sounds more general):

Example: Hom_G(X₁, X₂) (Guillemin-Sternberg 1982)

This is

 $Hom_G(X_1, X_2) := (X_1^- \times X_2) //G$

where (X_i, ω_i, Φ_i) are Hamiltonian G-spaces and $X^- := (X, -\omega, -\Phi)$ So the product here has diagonal G-action, 2-form $\omega_2 - \omega_1$, and moment map $\Phi(x_1, x_2) = \Phi_2(x_2) - \Phi_1(x_1)$.

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Note that ⁽²⁾ boils down to X₂//G when (X₁, ω₁, Φ₁) = ({0}, 0, 0);
 so asking when it carries a reduced 2-form includes the original question about .

More generally, Guillemin-Sternberg took for X_1 a coadjoint orbit: G(µ), and noted that \heartsuit then boils down to the space $\Phi_2^{-1}(\mu)/G_{\mu}$ of Marsden-Weinstein: this is their famous "shifting trick".

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D1) Covering. All constant maps $\mathbb{R}^n \to X$ are in \mathcal{P} , for all n.

Locality. Let V → X be a map with V ∈ x_n. If every point of V has an open neighborhood U such that P_{1U} ∈ P, then P ∈ P.

Smooth compatibility: Let $U \xrightarrow{\sim} V \xrightarrow{\sim} X$ be maps with $(U, V) \in X_{m} \times T_{p}$. If $P \in \mathcal{P}$ and $\psi \in C^{\infty}(U, V)$, then $P \circ \psi \in \mathcal{P}$.

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D2) *Locality.* Let $V \xrightarrow{P} X$ be a map with $V \in \tau_n$. If every point of V has an open neighborhood U such that $P_{|U} \in \mathcal{P}$, then $P \in \mathcal{P}$.

D3) Smooth compatibility. Let $U \xrightarrow{\psi} V \xrightarrow{P} X$ be maps with $(U, V) \in \tau_m \times \tau_n$. If $P \in \mathcal{P}$ and $\psi \in C^{\infty}(U, V)$, then $P \circ \psi \in \mathcal{P}$.

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Definitions

Let X be a set. A *diffeology* on X is a subset \mathcal{P} of $\bigcup_{n \in \mathbb{N}, U \in \tau_n} \operatorname{Maps}(U, X)$ satisfying (D1–D3). We call its members with domain $U \in \tau_n$, *n*-plots. A map $(X, \mathcal{P}) \xrightarrow{F} (Y, \Omega)$ of diffeological spaces (: sets with diffeologies) is called *smooth* if $P \in \mathcal{P}$ implies $F \circ P \in \Omega$.

If $(X, \mathcal{P}) \stackrel{\mu}{\rightarrow} (X, \Omega)$ is smooth, i.e. $\mathcal{P} \subset \Omega$, we call \mathcal{P} *finer* and Ω *coarser*.

 $\text{E.g.: } \{\text{locally constant maps}\} =: \mathbb{P}_{\text{discrete}} \subset \mathbb{P} \subset \mathbb{P}_{\text{coarse}} := \{\text{all maps}\}.$

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A map $(X, \mathcal{P}) \xrightarrow{F} (Y, \mathcal{Q})$ of diffeological spaces (: sets with diffeologies) is called *smooth* if $P \in \mathcal{P}$ implies $F \circ P \in \mathcal{Q}$.

If $(X, \mathcal{P}) \xrightarrow{id} (X, \Omega)$ is smooth, i.e. $\mathcal{P} \subset \Omega$, we call \mathcal{P} *finer* and Ω *coarser*.

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§6. Proper actions

§7. Frobeniu: reciprocity

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Y ∫i X

So every manifold has a canonical diffeology. But also:

Let Y be a diffeological space and *i* : X → Y an injection. Then X has a coarsest diffeology making *i* smooth, the *subset diffeology*. Its plots are the maps P : U → X such that *i* ∘ P is a plot of Y.

Universal property: A map F to X is smooth iff $i \circ$ F is smooth.

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Let X be a diffeological space and $s : X \to Y$ a surjection. Then Y has a finest diffeology making s smooth, the *quotient diffeology*. Its n-plots are the maps $Q : U \to Y$ that have around each $u \in U$ and $Q_W = s \circ R$.

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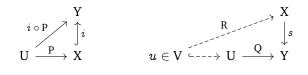
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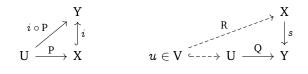
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Definitions

F is strict if both F and F⁻¹ are smooth (i.e., F is a diffeomorphism).
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§7. Frobenius reciprocity

• The promised *subquotient diffeology* of $X/\!/G = \Phi^{-1}(0)/G$ results: take subset diffeology on $\Phi^{-1}(0)$, then quotient — or *equivalently*, as one can show, take quotient diffeology on X/G, then subset.

• Any map $F:X \rightarrow Y$ between diffeological spaces can be factored

$$egin{array}{ccc} X & & \stackrel{F}{\longrightarrow} & Y \ s & & \uparrow i \ X/\sim & \stackrel{\dot{\mathrm{F}}}{\longrightarrow} & \mathrm{F}(\mathrm{X}), \end{array} egin{array}{ccc} \mathrm{F} & i \circ \dot{\mathrm{F}} \circ s, \ \mathrm{F} & i \circ \dot{\mathrm{F}} \circ s, \end{array}$$

where *s* = quotient map by the equivalence relation 'F(x_1) = F(x_2)', \dot{F} = bijection of that quotient with F(X), *i* = inclusion of that image into Y. With quotient (resp. subset) diffeology on X/~ (resp. F(X)), the universal properties we saw imply: F smooth \Leftrightarrow \dot{F} smooth.

Definitions

- F is *strict* if both \dot{F} and \dot{F}^{-1} are smooth (i.e., \dot{F} is a *diffeomorphism*).
- An *induction* is a strict injection. *Example*: inclusion of a subset.
- A subduction is a strict surjection. Example: projection to a quotient.

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§6. Proper actions

§7. Frobeniu reciprocity Let us call *ordinary* the *k*-forms on Euclidean open sets and operations on them (pull-back, exterior derivative).

Definitions (Diffeological Cartan-de Rham calculus) Let X and Y be diffeological spaces.

A b-form a on Y is a functional which sends each plot $P: Y \rightarrow S$ to an ordinary b-form on Y_i denoted P'a.

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A *k*-form α on Y is a functional which sends each plot P : V \rightarrow Y to an ordinary *k*-form on V, denoted P^{*} α . As compatibility, we require: if $\phi \in \mathbb{C}^{\infty}(U, V)$ (so P $\circ \phi$ is another plot), then

 $(\mathbf{P} \circ \phi)^* \alpha = \phi^* \mathbf{P}^* \alpha, \qquad \phi^*$: ordinary pull-back.

the pull-back \mathbb{P}^n by a smooth map $\mathbb{P}: \mathbb{K} \to \mathbb{Y}$ is the k-form on \mathbb{K} is the k-form on \mathbb{K} (so $\mathbb{P} \circ \mathbb{P}$ is a plot of \mathbb{Y}), then

 $\mathbb{P}^* \mathbb{P}^* \alpha = (\mathbb{P} \circ \mathbb{P})^* \alpha, \dots, \mathbb{P}^*$; being defined.

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Let us call *ordinary* the *k*-forms on Euclidean open sets and operations on them (pull-back, exterior derivative).

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Let X and Y be diffeological spaces.

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 $(\mathbf{P} \circ \boldsymbol{\psi})^* \boldsymbol{\alpha} = \boldsymbol{\psi}^* \mathbf{P}^* \boldsymbol{\alpha}, \qquad \boldsymbol{\psi}^* : \text{ ordinary pull-back.}$

Its *pull-back* $F^*\alpha$ by a smooth map $F : X \to Y$ is the *k*-form on X defined by: if P is a plot of X (so $F \circ P$ is a plot of Y), then

 $P^*F^*\alpha = (F \circ P)^*\alpha, \quad F^*$; being defined.

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 Its exterior derivative dα is the (k + 1)-form defined for all pl P of Y by P^{*} dα = dP^{*}α, with ordinary d on the right-hand side.

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§7. Frobeniu reciprocity

There is a basic criterion for when a k-form descends to a quotient:

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et $s : X \rightarrow Y$ be a subduction, α a k-form on X. In order that $\alpha = s^*\beta$ or some β on Y, it is necessary and sufficient that all pairs of plots P, Q f X satisfy

 $s \circ \mathbf{P} = s \circ \mathbf{Q} \qquad \Rightarrow \qquad \mathbf{P}^* \alpha = \mathbf{Q}^* \alpha.$

foreover, β is then unique.

Comments on the proof. Necessity is clear: if $\alpha = s^*\beta$, we have

$$\begin{split} \mathbf{P}^* \alpha &= \mathbf{P}^* s^* \beta = (s \circ \mathbf{P})^* \beta, \\ \mathbf{Q}^* \alpha &= \mathbf{Q}^* s^* \beta = (s \circ \mathbf{Q})^* \beta \end{split}$$

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by definition of s^* ; \diamond follows. Proving the rest takes about 2 pages. \Box

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All diffeological notions used in §1 have now been defined.

Existence of $\omega_{X/\!/G}$: Prior State of the Art

If the G-action on the level $C = \Phi^{-1}(0)$ is *locally free and proper*, it has long been known that X//G is an (effective) orbifold with an 'orbifold 2-form' (proof in Cushman-Bates 1997). Now when orbifolds are regarded as diffeological spaces, 'orbifold forms' define diffeological forms and conversely (Karshon-Watts 2016).

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 - Note: locally free means that the infinitesimal stabilizer g_{σ} is zero for all $x \in C$. As $im(D\Phi(x)) = annihilator(g_{\sigma})$, it follows that 0 is a negative value, so C is a manifold.
 - Of course, if the G-action on C is free and proper, then X/G itself (itself is a manifold with a symplectic 2-form (Manden-Weinstein X979)

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Of course, if the G-action on C is *free and proper*, then X/G itself is a manifold with a *symplectic* 2-form (Marsden-Weinstein 1974)

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- If the G-action on the level $C = \Phi^{-1}(0)$ is *locally free and proper*, it has long been known that X//G is an (effective) orbifold with an 'orbifold 2-form' (proof in Cushman-Bates 1997). Now when orbifolds are regarded as diffeological spaces, 'orbifold forms' define diffeological forms and conversely (Karshon-Watts 2016). So X//G carries a reduced 2-form in this case.
- Note: locally free means that the infinitesimal stabilizer g_x is zero for all x ∈ C. As Im(DΦ(x)) = annihilator(g_x), it follows that 0 is a regular value, so C is a manifold.
- Of course, if the G-action on C is *free and proper*, then X//G itself is a manifold with a *symplectic* 2-form (Marsden-Weinstein 1974).

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§4. Strict actions

Let a diffeological group G act on a diffeological space X. We consider the map

 $heta: \mathsf{G} imes \mathsf{X} o \mathsf{X} imes \mathsf{X}, \qquad heta(g,x) = (x,g(x)).$

Definition

The G-action is *strict* if θ is a strict map (§2).

Theorem 1

In the setting of §1, suppose that the G-action on $C = \Phi^{-1}(0)$ is strict. Then X//G carries a reduced 2-form.

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§4. Strict actions

§4. Strict actions

Let a diffeological group G act on a diffeological space X. We consider the map

> $\theta : \mathbf{G} \times \mathbf{X} \to \mathbf{X} \times \mathbf{X}.$ $\theta(q, x) = (x, q(x)).$

Definition

The G-action is *strict* if θ is a strict map (§2).

Theorem 1

In the setting of §1, suppose that the G-action on $C = \Phi^{-1}(0)$ is strict. *Then* X//G *carries* a *reduced* 2*-form*.

Comments on the proof. Strictness is "just what's needed" for a straightforward application of Souriau's criterion \Diamond . Subtler results

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Example: Ind_H^GY

This is

$Ind_{H}^{G}Y := (T^{*}G \times Y)/\!\!/H = \psi^{-1}(0)/H$

there: G is a Lie group, H is an *arbitrary subgroup* (hence canonically lso a Lie group: Bourbaki 1972), (Y, ω_Y, Ψ) is a Hamiltonian H-space, ad L := T*G × Y is the Hamiltonian G × H-space with action $(p, h)(p, y) = (gph^{-1}, h(y))$ and moment map $\phi \times \phi : L \to \mathfrak{g}^* \times \mathfrak{h}^*$,

 $egin{array}{rl} \phi(p,y) &=& pq^{-1} \ \psi(p,y) &=& \Psi(y) - q^{-1}p_{\parallel b} \end{array}$

- When H is closed, \clubsuit is a Marsden-Weinstein reduced manifold, with a residual G-action and moment map $\Phi_{1/dH}$: $Ind_{H}^{G}Y \rightarrow \mathfrak{g}^{*}$.
- When H is not closed, the H-action on ψ⁻¹(0) is still *strict*: so Theorem 1 yields a reduced 2-form ω_{L//H}, and we have a "para symplectic" *induced Hamiltonian G-space* (Ind^G_H Y, ω_{L//H}, Φ_{L//H})

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Example: Ind_H^GY

This is

$Ind_H^G\,Y:=(T^*G\times Y)/\!\!/H=\psi^{-1}(0)/H$

where: G is a Lie group, H is an *arbitrary subgroup* (hence canonically also a Lie group: Bourbaki 1972), (Y, ω_Y, Ψ) is a Hamiltonian H-space, and L := T*G × Y is the Hamiltonian G × H-space with action $(g, h)(p, y) = (gph^{-1}, h(y))$ and moment map $\phi \times \psi : L \to g^* \times \mathfrak{h}^*$, $\begin{pmatrix} \phi(p, y) = pq^{-1} \\ h(y) = pq^{-1} \end{pmatrix}$ $(p \in T_1^*G)$.

- When H is closed,
 is a Marsden-Weinstein reduced manifold, with a residual G-action and moment map Φ_{L/H} : Ind^G_HY → g^{*}.
- When H is not closed, the H-action on φ⁻¹(0) is still *strict*: so Theorem 1 yields a reduced 2-form ω_{L//H}, and we have a "para symplectic" *induced Hamiltonian G-space* (Ind^G_H Y, ω_{L//H}, Φ_{L//H})

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- When H is closed, ♣ is a Marsden-Weinstein reduced manifold, with a residual G-action and moment map Φ_{L/H} : Ind^G_HY → g*.
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- When H is closed, \clubsuit is a Marsden-Weinstein reduced manifold with a residual G-action and moment map Φ_{torn} : Ind^G $X \to a^*$
- When H is not closed, the H-action on ψ⁻¹(0) is still *strict*: so Theorem 1 yields a reduced 2-form ω_{L/H}, and we have a "para symplectic" *induced Hamiltonian G-space* (Ind^G_H Y, ω_{L/H}, Φ_{L/H})

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Example: Ind_H^GY

This is

$$Ind_{H}^{G} Y := (T^{*}G \times Y) / H = \psi^{-1}(0) / H$$

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 $egin{array}{rlll} & \phi(p,y) &=& pq^{-1} \ & \psi(p,y) &=& \Psi(y) - q^{-1}p_{\,|\,\mathfrak{h}} \end{array}$

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- When H is not closed, the H-action on ψ⁻¹(0) is still *strict*: so Theorem 1 yields a reduced 2-form ω_{L//H}, and we have a "para symplectic" *induced Hamiltonian G-space* (Ind^G_H Y, ω_{L//H}, Φ_{L//H})

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 $(g,h)(p,y) = (gph^{-1},h(y))$ and moment map $\phi \times \psi : L \to \mathfrak{g}^* \times \mathfrak{h}^*$,

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where: G is a Lie group, H is an arbitrary subgroup (hence canonically also a Lie group: Bourbaki 1972), (Y, ω_Y, Ψ) is a Hamiltonian H-space, and $L := T^*G \times Y$ is the Hamiltonian $G \times H$ -space with action $(q,h)(p,y) = (qph^{-1}, h(y))$ and moment map $\phi \times \psi : L \to \mathfrak{g}^* \times \mathfrak{h}^*$,

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- When H is closed, ♣ is a Marsden-Weinstein reduced manifold, with a residual G-action and moment map Φ_{L//H} : Ind^G_HY → g*.
- When H is not closed, the H-action on ψ⁻¹(0) is still *strict*: so Theorem 1 yields a reduced 2-form ω_{L//H}, and we have a "para symplectic" *induced Hamiltonian G-space* (Ind^G_H Y, ω_{L/H}, Φ_{L/H}

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- When H is not closed, the H-action on ψ⁻¹(0) is still *strict*: so Theorem 1 yields a reduced 2-form ω_{L//H}, and we have a "parasymplectic" *induced Hamiltonian G-space* (Ind_H^GY, ω_{L//H}, Φ_{L//H})

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- When H is not closed, the H-action on $\psi^{-1}(0)$ is still *strict*: so Theorem 1 yields a reduced 2-form $\omega_{L/\!/H}$, and we have a "parasymplectic" *induced Hamiltonian G-space* (Ind^G_H Y, $\omega_{L/\!/H}$, $\Phi_{L/\!/H}$).

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§7. Frobenius reciprocity

Instructive special case: Ind_H^G{0}

This is $(T^*G)/\!\!/H$, the reduction of T^*G by the 'right' action of H.

When H is closed, it is well known (Kummer-Marsden-Satzer) that $(T^*G)//H = T^*(G/H)$ with its canonical 2-form and G-action. When H is not closed, Iglesias-Zemmour (2010) gave meaning to the right-hand side by defining, for any diffeological space X, a "cotangent space" $T^*(X)$ with a canonical 2-form d Liouv and Hamiltonian action of Diff(X).

We can ask, then, if the equality survives. It does at least for dense H:

Theorem 2

Let G be a Lie group, H any dense subgroup. Then $(T^*G)/H = T^*(G/H)$ as diffeological, parasymplectic Hamiltonian G-spaces.

Example: G the 2-torus, H an irrational winding, $G/H = T_{\alpha}$

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Let G be a Lie group, H any dense subgroup. Then $(T^*G)/H = T^*(G/H)$ as diffeological, parasymplectic Hamiltonian G-spaces.

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Instructive special case: Ind_H^G{0}

This is $(T^*G)//H$, the reduction of T^*G by the 'right' action of H.

- When H is closed, it is well known (Kummer-Marsden-Satzer) that $(T^*G)/\!\!/H = T^*(G/H)$ with its canonical 2-form and G-action.
- When H is not closed, Iglesias-Zemmour (2010) gave meaning to the right-hand side by defining, for any diffeological space X, a "cotangent space" T*(X) with a canonical 2-form *d* Liouv and Hamiltonian action of Diff(X).

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In the setting of §1, suppose the G-action on $C = \Phi^{-1}(0)$ is locally free (i.e. all $x \in C$ have infinitesimal stabilizer $g_x = \{0\}$) and G is connected. Then X/G carries a reduced 2-form.

Comments on the proof. Under the hypotheses, standard properties of the moment map:

 $\operatorname{Ker}(\operatorname{D}\Phi(x)) = \mathfrak{g}(x)^{\omega}, \qquad \operatorname{Im}(\operatorname{D}\Phi(x)) = \operatorname{annihilator}(\mathfrak{g}_x)$

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readily imply that 1) 0 is a regular value of Φ , 2) C is a submanifold,

3) in C the G-orbits are the leaves of a *foliation* \mathcal{F} , 4) $\omega_{|C} := j^* \omega$ is *basic* for \mathcal{F} , i.e., G-invariant with $\mathfrak{g}(x) \subset \operatorname{Ker}(\omega_{|C})$. A theorem on foliations by Hector *et al.* (2011) then implies the result.

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Recall that the action of a Lie group on a manifold is called *proper* if the map θ (§4) is proper, i.e., compact sets have compact preimages.

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In the setting of §1, suppose that the G-action on X is **proper**. (Example: G compact.) Then X//G carries a reduced 2-form ω_{X/G}.

Comments on the proof. For proper actions, Sjamaar-Lerman-Bates (1991, 1997) showed that X//G = C/G is a 'stratified symplectic space', i.e. (among other things) a disjoint union of symplectic manifolds $(C_t/G, \omega_t)$ indexed by orbit types t. Our proof crucially uses the ω_t to show that $j^*\omega$ satisfies Souriau's criterion \Diamond . The resulting (global) $\omega_{X/G}$ actually induces every ω_t , as the following corollary states.

Corollary

In Theorem 4, $\omega_{X/\!/G}$ restricts to the Sjamaar-Lerman-Bates ω_t on each reduced piece C_t/G .

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Let G be a Lie group, H a closed subgroup, X a Hamiltonian G-space, Y a Hamiltonian H-space. Recall Hom_G and Ind_H^G , and define Res_H^G .

here is a (diffeological) diffeomorphism

 $t: \operatorname{Hom}_{G}(X, \operatorname{Ind}_{\operatorname{H}}^{\operatorname{G}} Y) \to \operatorname{Hom}_{\operatorname{H}}(\operatorname{Res}_{\operatorname{H}}^{\operatorname{G}} X, Y).$

Moreover, if one side carries a reduced 2-form, then so does the other, and t maps one form to the other.

Sketch of proof. The sides are respectively (M//H)//G and N//H, where

 $M = X^- \times T^*G \times Y$, resp. $N = X^- \times Y$

have $G \times H$ -action $(g, h)(x, p, y) = (g(x), gph^{-1}, h(y))$, resp. diagonal H-action, plus appropriate 2-forms ω_M and ω_N and moment maps

 $\phi_{M} \times \phi_{M} : M \to \mathfrak{g}^{*} \times \mathfrak{h}^{*}, \quad \text{resp.} \quad \phi_{N} : N \to \mathfrak{h}^{*}.$ Define $r : M \to N$ by $r(x, p, y) = (q^{-1}(x), y)$ for $p \in T_{\sigma}^{*}G$,

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have $G \times H$ -action $(g, h)(x, p, y) = (g(x), gph^{-1}, h(y))$, resp. diagonal H-action, plus appropriate 2-forms ω_M and ω_N and moment maps

 $\phi_{\mathrm{M}} \times \phi_{\mathrm{M}} : \mathrm{M} \to \mathfrak{g}^* \times \mathfrak{h}^*, \quad \text{resp.} \quad \phi_{\mathrm{N}} : \mathrm{N} \to \mathfrak{h}^*.$ Define $r : \mathrm{M} \to \mathrm{N}$ by $r(x, p, y) = (q^{-1}(x), y)$ for $p \in \mathrm{T}^*_{\sigma}\mathrm{G}$,

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§3. Orbifolds

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§5. Locally free actions

§6. Proper actions

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§7. Frobenius reciprocity

Let G be a Lie group, H a closed subgroup, X a Hamiltonian G-space, Y a Hamiltonian H-space. Recall Hom_{G} and Ind_{H}^{G} , and define Res_{H}^{G} .

Theorem **S**

here is a (diffeological) diffeomorphism

 $t: \operatorname{Hom}_{G}(X, \operatorname{Ind}_{H}^{G} Y) \to \operatorname{Hom}_{H}(\operatorname{Res}_{H}^{G} X, Y).$

Moreover, if one side carries a reduced 2-form, then so does the other, and t maps one form to the other.

Sketch of proof. The sides are respectively (M//H)//G and N//H, where

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§7. Frobenius reciprocity

and consider the commutative diagram



§2. Diffeology

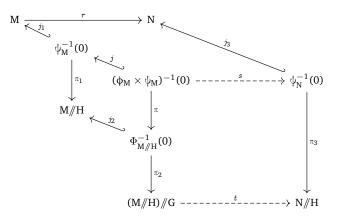
§3. Orbifolds

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where the *j*'s and π 's are inclusions and projections as in §1. One checks that *r* sends $(\phi_M \times \psi_M)^{-1}(0)$ to $\psi_N^{-1}(0)$, so there is a map *s* as indicated; and *s* sends G × H-orbits to H-orbits, so there is a map *t*. Likewise one checks that the right inverse $r' : N \to M$ defined by

§7. Frobenius reciprocity

and consider the commutative diagram



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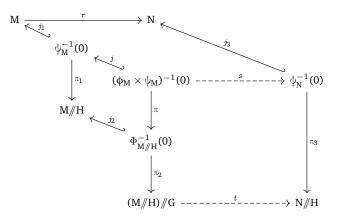
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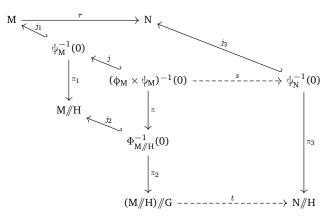
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and consider the commutative diagram



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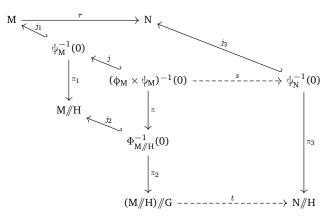
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§7. Frobenius reciprocity

and consider the commutative diagram



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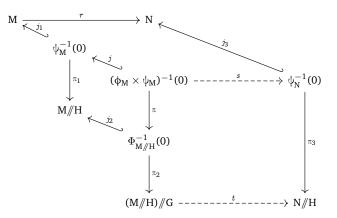
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§7. Frobenius reciprocity

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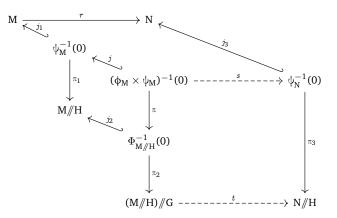
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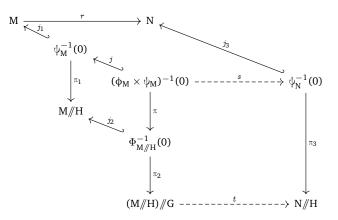
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 $r'(x, y) = (x, \Phi(x), y)$ (where we identify $\mathfrak{g}^* \cong T_e^* G$) descends to an inverse t^{-1} of t. Now r and r' are quite explicitly smooth. Using the universal properties of inductions (the j's) and subductions (the π 's), one deduces without trouble that t and t are smooth, as claimed

Next, assume that both sides carry reduced 2-forms, $\omega_{(M/H)//G}$ and $\omega_{N//H}$. We must prove $\omega_{(M//H)//G} = t^* \omega_{N//H}$. By chasing the diagram, one checks that this is equivalent to, not quite $\omega_M = r^* \omega_N$ but

$$j^* j_1^* \omega_{\rm M} = j^* j_1^* r^* \omega_{\rm N},$$
 (*)

an equality of 2-forms on $(\phi_M \times \phi_M)^{-1}(0)$ (usually not a manifold). Now (*) means that its sides coincide after pull-back by any plot P of that subset, i.e., by any *smooth map* P : U \rightarrow M *taking values in* $(\phi_M \times \phi_M)^{-1}(0)$. This is true, and can be checked in about 10 lines. Finally, assume merely that *one* reduced form exists, $\omega_{(M/H)/G}$ or

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reduction §2. Diffeology

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easy chase using again (*) shows that it is indeed a reduced form.

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 $\omega_{\text{N/H}}$. Then we can *define* the other by $\omega_{(\text{M/H})/G} = t^* \omega_{\text{N/H}}$; and an easy chase using again (*) shows that it is indeed a reduced form.

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 $\omega_{\text{N/H}}$. Then we can *define* the other by $\omega_{(\text{M/H})/\text{G}} = t^* \omega_{\text{N/H}}$; and an easy chase using again (*) shows that it is indeed a reduced form.

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 $r'(x, y) = (x, \Phi(x), y)$ (where we identify $\mathfrak{g}^* \cong T_e^*G$) descends to an inverse t^{-1} of t. Now r and r' are quite explicitly smooth. Using the universal properties of inductions (the j's) and subductions (the π 's), one deduces without trouble that t and t^{-1} are smooth, as claimed.

Next, assume that both sides carry reduced 2-forms, $\omega_{(M/H)//G}$ and $\omega_{N//H}$. We must prove $\omega_{(M/H)//G} = t^* \omega_{N//H}$. By chasing the diagram, one checks that this is equivalent to, not quite $\omega_M = r^* \omega_N$ but

$$j^* j_1^* \omega_{\mathrm{M}} = j^* j_1^* r^* \omega_{\mathrm{N}},$$
 (*

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$$j^* j_1^* \omega_{\rm M} = j^* j_1^* r^* \omega_{\rm N},$$
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Next, assume that both sides carry reduced 2-forms, $\omega_{(M/H)//G}$ and $\omega_{N//H}$. We must prove $\omega_{(M//H)//G} = t^* \omega_{N//H}$. By chasing the diagram, one checks that this is equivalent to, not quite $\omega_M = r^* \omega_N$ but

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Next, assume that both sides carry reduced 2-forms, $\omega_{(M/H)//G}$ and $\omega_{N//H}$. We must prove $\omega_{(M//H)//G} = t^* \omega_{N//H}$. By chasing the diagram, one checks that this is equivalent to, not quite $\omega_M = r^* \omega_N$ but

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 $\omega_{N/\!/H}$. Then we can *define* the other by $\omega_{(M/\!/H)/\!/G} = t^* \omega_{N/\!/H}$; and an easy chase using again (*) shows that it is indeed a reduced form.

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Final remark: Everything we have said can be adapted to work also in the category {*prequantum G-spaces*}, which more closely mirrors the motivating category {unitary representations}. For details, see arXiv:2007.9434 and arXiv:2403.3927.

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End!