Motivation

Symplection induction

Mackey-Wigner

Kirillov-Bernat

Imprimitivity

Mackey theory

Contact imprimitivity

Symplectic Mackey Theory*

François Ziegler (Georgia Southern)

Shanghai Jiao Tong University May 17, 2018

Abstract: Guillemin & Sternberg used symplectic induction to give a "Mackey-Wigner" description of Hamiltonian G-spaces when G has a *normal abelian semidirect factor* N. I will describe how this generalizes to a full "Mackey" description (and classification) valid for *arbitrary normal* subgroups N, and explain why this is best done in the setting of *prequantum* (contact) G-spaces.

^{*}http://arxiv.org/abs/1410.7950

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$$1 \longrightarrow N \longrightarrow G \longrightarrow G/N \longrightarrow 1.$$

Questions about G often reduce to similar ones about N and G/N.



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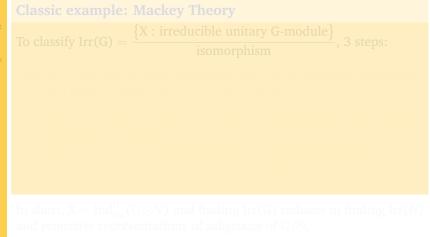
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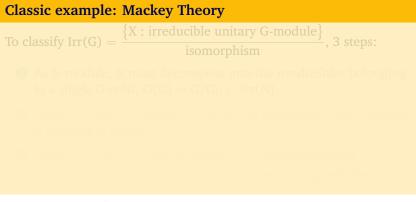
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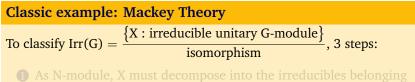
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Classic example: Mackey Theory To classify $Irr(G) = \frac{\{X : irreducible unitary G-module\}}{isomorphism}$, 3 steps:

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Symplectic induction (G: Lie group)

Given a closed subgroup $H \subset G$ and a hamiltonian H-space (Y, τ, Ψ) , Kazhdan-Kostant-Sternberg (1978) produce a hamiltonian G-space

 $(\operatorname{Ind}_{\operatorname{H}}^{\operatorname{G}} \operatorname{Y}, \sigma_{\operatorname{ind}}, \Phi_{\operatorname{ind}})$

as follows:

Endow M := T*G × Y with the 2-form ω = dθ + τ, θ = "(p, dq)".
Let H act on M by h(p, y) = (ph⁻¹, h(y)).
This has moment map ψ(p, y) = Ψ(y) - q⁻¹p₁, (p ∈ T_q^{*}G).
Define Ind₁⁽¹⁾ X := ψ⁻¹(0)/H. (Marsden-Weinstein subquotient).
The G-action g(p, y) = (gp, y) and moment map φ(p, y) = pq⁻¹ pass to the quotient; whence the claimed G-space structure.

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as follows:

- **1** Endow $M := T^*G \times Y$ with the 2-form $\omega = d\theta + \tau$, $\theta = (\langle p, dq \rangle)^*$.
- 2 Let H act on M by $h(p, y) = (ph^{-1}, h(y))$.
- 3 This has moment map $\psi(p, y) = \Psi(y) q^{-1}p_{|\mathfrak{h}|}$ $(p \in \mathrm{T}_q^*\mathrm{G}).$

Define $\operatorname{Ind}_{H}^{G} Y := \psi^{-1}(0)/H$ (Marsden-Weinstein subquotient).

Solution The G-action g(p, y) = (gp, y) and moment map $\varphi(p, y) = pq^{-1}$ pass to the quotient; whence the claimed G-space structure.

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Proposition (Elementary properties)

 $\mathbf{D} \operatorname{dim}(\operatorname{Ind}_{\operatorname{H}}^{\operatorname{G}} Y) = 2 \operatorname{dim}(\operatorname{G}/\operatorname{H}) + \operatorname{dim}(Y).$

 $\operatorname{M} \operatorname{meets} \operatorname{Im}(\Phi_{\operatorname{ind}}) \Leftrightarrow \operatorname{M}_{|\mathfrak{h}} \operatorname{meets} \operatorname{Im}(\Psi) \quad (\mathfrak{M} \in \mathfrak{g}^*/\mathsf{G}) \text{ (Frobenius).}$

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Symplectic induction (G: Lie group)

Proposition (Elementary properties)

1 dim(Ind_H^G Y) = $2 \dim(G/H) + \dim(Y)$.

2 M meets $\operatorname{Im}(\Phi_{\operatorname{ind}}) \Leftrightarrow M_{|\mathfrak{h}}$ meets $\operatorname{Im}(\Psi)$ ($M \in \mathfrak{g}^*/G$) (Frobenius).

 $\operatorname{Ind}_{H}^{G} Y$ is a coadjoint orbit $\Rightarrow Y$ is a coadjoint orbit.

 $\operatorname{Ind}_{K}^{G}\operatorname{Ind}_{H}^{K}Y = \operatorname{Ind}_{H}^{G}Y$ (K: intermediate closed subgroup) (Stages).

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Proposition (Elementary properties)

- $(Ind_{H}^{G} Y) = 2 \dim(G/H) + \dim(Y).$
- 2 M meets $\operatorname{Im}(\Phi_{\operatorname{ind}}) \Leftrightarrow M_{|\mathfrak{h}}$ meets $\operatorname{Im}(\Psi) \quad (M \in \mathfrak{g}^*/G)$ (Frobenius).
 - 3) $\operatorname{Ind}_{\operatorname{H}}^{\operatorname{G}}\operatorname{Y}$ is homogeneous \Rightarrow Y is homogeneous.
 - Ind^G_H Y is a coadjoint orbit \Rightarrow Y is a coadjoint orbit.
 - Ind^G_K Ind^K_H Y = Ind^G_H Y (K: intermediate closed subgroup) (Stages).

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- $(Ind_H^G Y) = 2 \dim(G/H) + \dim(Y).$
- 2 M meets $Im(\Phi_{ind}) \Leftrightarrow M_{|\mathfrak{h}}$ meets $Im(\Psi) \quad (M \in \mathfrak{g}^*/G)$ (Frobenius).
- **3** $Ind_{H}^{G} Y$ is homogeneous $\Rightarrow Y$ is homogeneous.
- 4 $\operatorname{Ind}_{\operatorname{H}}^{\operatorname{G}} Y$ is a coadjoint orbit \Rightarrow Y is a coadjoint orbit.
- **5** $\operatorname{Ind}_{K}^{G}\operatorname{Ind}_{H}^{K}Y = \operatorname{Ind}_{H}^{G}Y$ (K: *intermediate closed subgroup*) (*Stages*).

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Theorem (Guillemin-Sternberg 1983++)

Let $N \subset G$ be a closed connected normal **abelian** subgroup. Pick $u \in \mathfrak{n}^*$ and write $H = G_u$. Then $Y \mapsto X = Ind_H^G Y$ defines a bijection between

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(a) coadjoint orbits X of G such that $X_{|n} \supset \{u\}$;

b) coadjoint orbits Y of H such that $Y_{|n} = \{u\}$.

he inverse map sends X = G(x) to $Y = H(x_{|b})$

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The inverse map sends X = G(x) to $Y = H(x_{|\mathfrak{h}}) \cong (x \mapsto x_{|\mathfrak{n}})^{-1}(u)/N$ $(x_{|\mathfrak{h}} = u)$. Note that Y is a homogeneous symplectic manifold of H/N.

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Crux of proof. Suppose $Y = H(x_{|\mathfrak{h}})$. Clearly $Y_{|\mathfrak{n}} = \{u\}$. Want: Φ_{ind} 1-1 onto X. Now, we have

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Let $N \subset G$ be a closed connected normal **abelian** subgroup. Pick $u \in \mathfrak{n}^*$ and write $H = G_u$. Then $Y \mapsto X = Ind_H^G Y$ defines a bijection between

- (a) coadjoint orbits X of G such that $X_{|n} \supset \{u\}$;
- (b) coadjoint orbits Y of H such that $Y_{|n} = \{u\}$.

The inverse map sends X = G(x) to $Y = H(x_{|\mathfrak{h}}) \cong (x \mapsto x_{|\mathfrak{n}})^{-1}(u)/N$ $(x_{|\mathfrak{n}} = u)$. Note that Y is a homogeneous symplectic manifold of H/N.

Crux of proof. Suppose $Y = H(x_{|\mathfrak{h}})$. Clearly $Y_{|\mathfrak{n}} = \{u\}$. Want: Φ_{ind} 1-1 onto X. Now, we have

(1) $\mathfrak{n}(x) = \operatorname{ann}(\mathfrak{h}).$

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(1) $\mathfrak{n}(x) = \operatorname{ann}(\mathfrak{h})$. Indeed: $\langle \mathfrak{n}(x), Z \rangle = \langle x, [\mathfrak{n}, Z] \rangle = \langle u, [\mathfrak{n}, Z] \rangle = \langle Z(u), \mathfrak{n} \rangle$ shows $\operatorname{ann}(\mathfrak{n}(x)) = \mathfrak{g}_u = \mathfrak{h}$.

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- (1) $\mathfrak{n}(x) = \operatorname{ann}(\mathfrak{h}).$
- (2) N(x) = x + ann(h). Indeed, for Z \in n: $\langle \exp(Z)(x), Z' \rangle = \langle x, \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \operatorname{ad}(Z)^n (Z') \rangle$ = $\langle x, Z' - [Z, Z'] \rangle = \langle x + Z(x), Z' \rangle$. So N(x) $\supset x + \mathfrak{n}(x)$.

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(2)
$$N(x) = x + \operatorname{ann}(\mathfrak{h}) = \eta^{-1}(x_{|\mathfrak{h}})$$
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- (3) $H(x) = \eta^{-1}(Y)$.

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- (3) $H(x) = \eta^{-1}(Y)$.

So $M_{|\mathfrak{h}}$ meets $Y \Rightarrow M$ meets X, and $Im(\Phi_{ind}) = X$ by Frobenius.

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Example: Poincaré orbits

Conside

$$\mathrm{G}=\left\{g=egin{pmatrix}\mathrm{L}&\mathrm{C}\0&1\end{pmatrix}\colon egin{pmatrix}\mathrm{L}\in\mathbf{SO}(3,1)^{\mathrm{o}}\C\in\mathbf{R}^{3,1}\end{array}
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a) X_{1n} is half a timelike hyperboloid and Y a coadjoint orbit of SO()
 b) X_{1n} is a half-cone and Y a coadjoint orbit of B(2)

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(a) $X_{|n|}$ is half a timelike hyperboloid and Y a coadjoint orbit of SO(3)

(b) $X_{|n|}$ is a half-cone and Y a coadjoint orbit of **E**(2)

(c) $X_{|n|}$ is a spacelike hyperboloid and Y a coadjoint orbit of **SL**(2, **R**)

(d) $X_{|n|}$ is the origin and Y(=X) a coadjoint orbit of $SO(3,1)^{\circ}$.

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(a) $X_{|n|}$ is half a timelike hyperboloid and Y a coadjoint orbit of ${f SO}(3)$

(b) $X_{|\mathfrak{n}}$ is a half-cone and Y a coadjoint orbit of E(2)

(c) $X_{|n|}$ is a spacelike hyperboloid and Y a coadjoint orbit of SL(2, R)

(d) $X_{|n|}$ is the origin and Y(=X) a coadjoint orbit of **SO**(3, 1)°.

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(a) $X_{\mid \mathfrak{n}}$ is half a timelike hyperboloid and Y a coadjoint orbit of $\boldsymbol{SO(3)}$

- (b) $X_{|\mathfrak{n}}$ is a half-cone and Y a coadjoint orbit of E(2)
- (c) $X_{|n|}$ is a spacelike hyperboloid and Y a coadjoint orbit of SL(2, R)
- (d) $X_{|n|}$ is the origin and Y(=X) a coadjoint orbit of **SO**(3, 1)^o.

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Application: Symplectic Kirillov-Bernat theory

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Theorem (Z.)

Let G be an exponential Lie group (: exp is a diffeomorphism $\mathfrak{g} \to G$) and let X = G(x) be a coadjoint orbit of G. Then X is monomial, i.e. G admits a closed connected subgroup H, such that

 $X = Ind_{H}^{G} \{ x_{|\mathfrak{h}} \}.$

Sketch of proof. A lemma of Takenouchi (1957) ensures that $\mathfrak{g}/\operatorname{ann}(X)$ admits an abelian ideal which is not central. Its preimage \mathfrak{n} in \mathfrak{g} is an X-abelian ideal which is not X-central. So the theorem gives $X = \operatorname{Ind}_{G_1}^G X_1$ where G_1 is the stabilizer of $\mathfrak{a}_{|\mathfrak{n}|}$ and $X_1 = G_1(\mathfrak{a}_{|\mathfrak{g}|})$. One checks that G_1 is again exponential and of smaller dimension than G. So we can iterate to obtain decreasing G_1 such that

 $\mathrm{X} = \mathrm{Ind}_{\mathrm{G}_1}^{\mathrm{G}} \cdots \mathrm{Ind}_{\mathrm{G}_i}^{\mathrm{G}_{i-1}} \mathrm{X}_i = \mathrm{Ind}_{\mathrm{G}_i}^{\mathrm{G}} \mathrm{X}_i$

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Sketch of proof. A lemma of Takenouchi (1957) ensures that $\mathfrak{g}/\operatorname{ann}(X)$ admits an abelian ideal which is not central. Its preimage \mathfrak{n} in \mathfrak{g} is an X-abelian ideal which is not X-central. So the theorem gives $X = \operatorname{Ind}_{G_1}^G X_1$ where G_1 is the stabilizer of $x_{|n|}$ and $X_1 = G_1(x_{|\mathfrak{g}_1})$. One checks that G_1 is again exponential and of smaller dimension than G. So we can iterate to obtain decreasing G_1 such that

 $\mathbf{X} = \mathrm{Ind}_{\mathrm{G}_{1}}^{\mathrm{G}} \cdots \mathrm{Ind}_{\mathrm{G}_{i}}^{\mathrm{G}_{i-1}} \mathbf{X}_{i} = \mathrm{Ind}_{\mathrm{G}_{i}}^{\mathrm{G}} \mathbf{X}_{i}$

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Application: Symplectic Kirillov-Bernat theory

Theorem (Z.)

Let G be an exponential Lie group (: exp is a diffeomorphism $\mathfrak{g} \to G$) and let X = G(x) be a coadjoint orbit of G. Then X is monomial, i.e. G admits a closed connected subgroup H, such that

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Let G be a locally compact group (e.g. Lie), X a unitary G-module.

Definition

A system of imprimitivity for X is a G-invariant, commutative \mathbb{T} -subalgebra $A \subset End(X)$.

Its base is its Gelfand spectrum $\mathbb{B} = \{nonzero \ laboration philometry of pointwise convergence.$

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Remark: The *Gelfand transform* $a \mapsto \hat{a}$, defined by $\hat{a}(b) = b(a)$, is an isomorphism $A \to C_0(B)$. Its inverse E is a *-representation of $C_0(B)$ in X such that

$$\mathrm{E}(f\circ g_{\mathrm{B}}^{-1})=g_{\mathrm{X}}\mathrm{E}(f)g_{\mathrm{X}}^{-1},$$

i.e. a "system of imprimitivity" in the original Mackey-Blattner sense.

Imprimitivity (motivation)

The point of this is:

Theorem (Frobenius, Mackey)

The following are equivalent:

- X admits a transitive system of imprimitivity with base B = G/H (H = G_b say);
 - $X = Ind_H^G Y$ for some unitary H-module Y (suitably unique).

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Explanation (case G/H admits a G-invariant measure):

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Explanation (case G/H admits a G-invariant measure):

- \Downarrow : Harder!

Symplectic imprimitivity

Let (X, σ, Φ) be a hamiltonian G-space.

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Contact imprimitivity

Let (X, σ, Φ) be a hamiltonian G-space.

Definition

A system of imprimitivity for X is a G-invariant, commutative Lie subalgebra $\mathfrak{f} \subset \mathbb{C}^{\infty}(X)$, such that the hamiltonian vector field drag f is complete for all $f \in \mathfrak{f}$.

Its *base* is the image B of the "moment map" $\pi : X \to f^*$, $(\pi(x), f) = f(x)$. Each $f \in f$ descends to a function f on B. The base, B, is a G-subset of f^* : $(q_0(b), f) = (b, f \circ q_0)$.

The system, f, is called transitive if $3^{(0)}$ G acts transitively on Bir 2°) $n : (X \rightarrow B)$ is C⁽ⁿ⁾ for the homogeneous space structure on B.

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- Kirillov-Bernat
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Explanation: $\mathcal{F} := (\mathfrak{f} \text{ as an additive group) acts on X by <math>f_X = e^{\operatorname{drag} f}$ and π is *formally* a moment map for this action: $\operatorname{drag}\langle \pi(\cdot), f \rangle = \operatorname{drag} f$. Stabilizers G_b are *closed* so B's homogeneous structure is well-defined.

Symplectic imprimitivity theorem

Theorem (Z.)

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Contact imprimitivity The following are equivalent for a hamiltonian G-space (X, σ, Φ) :

 X admits a transitive system of imprimitivity with base B = G/H (H = G_b say);

• $X = Ind_H^G Y$ for a hamiltonian H-space (Y, τ, Ψ) (suitably unique).

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Explanation:

↑: Recall, $Ind_{H}^{G}Y = (T^{*}G \times Y)//H$. Now a G-equivariant projection

 $\pi_{ind}:Ind_{H}^{G}\,Y\to G/H$

arises by noting that the map $T^*G \times Y \to G/H$ sending $T^*_qG \times Y$ to qH is constant on H-orbits, hence passes to the (sub)quotient. Then one checks that

 $\mathfrak{f}_{ind}:=\pi^*_{ind}(C^\infty(G/H))$

is a transitive system of imprimitivity on $Ind_H^G Y$ with base G/H.

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Explanation:

↓: Formally this is Mackey-Wigner applied to the group 𝔅 ⋊ G and abelian normal subgroup 𝔅. Explicitly Y is the "reduced space" π⁻¹(b)/𝔅. Proof subtler as 𝔅 need not be Lie, nor its action free or proper...

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From now on, suppose $N \subset \mathsf{G}$ is a closed normal subgroup. Then

- G acts naturally on N and n by conjugation.
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$$\begin{array}{ccc} X & & & \\ & & & \\ & & \\ G & & & \\ & & & & \\ & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & &$$

Little group step

Hence this triviality (where G_U or G_U/N is known as the *little group*):

Theorem

(*) maps any homogeneous hamiltonian G-space (Χ, σ, Φ) onto some G-orbit B = G(U) = G/G_U in n*/N.

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$$\begin{array}{cccc} X & \longrightarrow & \mathfrak{g}^* & \longrightarrow & \mathfrak{n}^* & \longrightarrow & \mathfrak{n}^* / N. \\ & & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & &$$

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$$X \xrightarrow{\Phi} \mathfrak{g}^* \xrightarrow{(\cdot)_{|\mathfrak{n}}} \mathfrak{g}^* \xrightarrow{N(\cdot)} \mathfrak{n}^* / N.$$

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The inverse map sends X to the Kazhdan-Kostant-Sternberg reduced space of X at U: the quotient of $\Phi(\cdot)_{|\mathfrak{n}|}^{-1}(U)$ by its characteristic foliation. Moreover

X is a coadjoint orbit of $\mathsf{G}\ \Longleftrightarrow\ Y$ is a coadjoint orbit of H.

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this action is symplectic, but a moment ψ : U → g_U^{*} need not exist;
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a certain cover Ğ_U [⊥]→ G_U acts, but ψ need not be Ğ_U-equivariant;
a cocycle θ_U exists such that ψ(ğ(ū)) = g(ψ(ū)) + θ_U(ğN); (*)
whence a *Mackey obstruction* class [θ_U] ∈ H¹(Ğ_U/N, (g_U/n)*).
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More details. \tilde{U} and \tilde{G}_U are built as follows: Fix a $c \in U$ once and for all, and write N_c° for the identity component of the stabilizer N_c .

where $\Delta(k) = (k^{-1}, k)$ and $\pi(n, l) = nl$:



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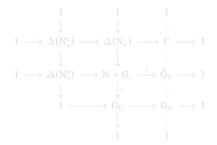
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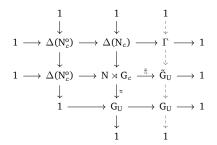
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Putting the last 3 theorems together, we get:

Theorem (Z.)

Let (X, σ, Φ) be a homogeneous hamiltonian G-space and $N \subset G$ a closed normal subgroup. Then $\Phi(X)$ sits above a G-orbit G(U) in \mathfrak{n}^*/N . If the tabilizer G_U is closed and U = N(c) has Mackey obstruction [θ], then here is a unique hamiltonian (G_U/N , [$-\theta$])-space (V, ω, φ) such that

$\mathrm{X} = \mathrm{Ind}_{\mathrm{G}_{\mathrm{U}}}^{\mathrm{G}} (ilde{\mathrm{U}} imes_{\Gamma} \mathrm{V})$.

where $\tilde{U} = N/N_c^0$ is the covering of U with group $\Gamma = N_c/N_c^0$. Every homogeneous hamiltonian ($\tilde{G}_U/N, [-\theta]$)-space V arises in this way.





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Contact imprimitivity theorem

Trying to mimick Mackey analysis for *nonabelian* normal subgroups forced us to remember that unitary representations really correspond to *prequantum* G-spaces (\tilde{X} , α).

These are principal circle bundles P : $\tilde{X} \to X$ with G-action preserving a connection 1-form α . The G-action, 2-form $d\alpha$, and 'contact' moment map $\langle \tilde{\Phi}(\tilde{x}), Z \rangle = \alpha(Z(\tilde{x}))$ descend and make X a hamiltonian G-space.

Suppose (\tilde{Y}, β) is a prequantum H-space over Y. Then (T*G × $\tilde{Y}, \theta + \beta$) is a prequantum G × H-space over T*G × Y. Reducing by H produces an *induced* prequantum G-space (Ind^G_H \tilde{Y}, α_{ind}) over Ind^G_H Y admitting a transitive *system of imprimitivity*: a G-invariant, commutative Lie algebra \tilde{f}_{ind} of complete α -preserving vector fields. In fact $\tilde{f}_{ind} \simeq f_{ind}$.

Theorem

Let (\tilde{X}, α) be a prequantum G-space. The following are equivalent:

• \tilde{X} admits a transitive system of imprimitivity with base B = G/H;

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Suppose (\tilde{Y},β) is a prequantum H-space over Y. Then $(T^*G \times \tilde{Y},\theta + \beta)$ is a prequantum $G \times H$ -space over $T^*G \times Y$. Reducing by H produces an *induced* prequantum G-space $(Ind_H^G \tilde{Y}, \alpha_{ind})$ over $Ind_H^G Y$ admitting a transitive *system of imprimitivity*: a G-invariant, commutative Lie algebra \tilde{f}_{ind} of complete α -preserving vector fields. In fact $\tilde{f}_{ind} \simeq f_{ind}$.

Theoren

Let (\tilde{X}, α) be a prequantum G-space. The following are equivalent:

- \tilde{X} admits a transitive system of imprimitivity with base B = G/H;
- $\tilde{X} = Ind_{H}^{G} \tilde{Y}$ for some prequantum H-space (\tilde{Y}, β) (suitably unique).

Motivation

Symplection induction

Mackey-Wigner

Kirillov-Bernat Imprimitivity Mackey theory Contact

Contact imprimitivity

Contact imprimitivity theorem

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End!